Effect of Cross-flow Momentum on Opposing Jet Mixing

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ABSTRACT
The flow of opposing jets was studied in computational fluid dynamics simulation. To clarify the flow field of the jet engine combustor, it is necessary to study air-jet dilution effects typical of opposing jets. The velocity distribution and turbulence intensity were obtained in large eddy simulation for air and an average Reynolds number of $2 \times 10^4$ corresponding to the jet diameter and velocity. Results obtained numerically generally agreed quantitatively with experimental results obtained in our previous study. Simulations were then carried out to clarify the effect of the momentum flux ratio $J$ ($4, 9, 16,$ and $64$) on mixing. Unmixedness was found to be highest for $J = 4$ since the penetrations and thus collision of opposing jets were weak for $J = 4$. When $J = 9, 16, 64$, mixing was improved by jet collision. It is proposed that the mixing mechanisms are differed depending on $J$.

INTRODUCTION
The operating temperatures of combustors are being raised each year to enhance the efficiency of gas turbine engines. It is desirable to reduce the size and mass of a combustor, yet reducing the combustor size negatively affects the temperature distribution at the exit of the combustor. Turbine blades can be damaged if the exhaust gas creates hot spots, which are areas of extremely high temperature. Thus, the temperature distribution at the combustor exit should be adjusted in the development of gas turbine engines. Figure 1 shows a common rich-burn quick-quench lean-burn combustor (Lefebvre and Ballal, 2010). In the fuel-rich primary zone, the injected fuel is burned with swirling air. Downstream of the primary zone, dilution air is required to complete the combustion process. In this type of combustor, the temperature distribution of the combustor is controlled with dilution air. It is thus important to discuss the details of the flow field.

Mixing of dilution air is an old problem that is usually studied using a simplified shape like that in Fig. 2. According to Holdeman et al. (1973), jet penetration and mixing are characterized by the hole shape, hole diameter, hole spacing, duct height, and momentum flux ratio.

The collision plane of the opposed jets is generally known to be unstable. Maki and Ogawa (1992) stated that the impact position of opposing jets is not fixed at a stable point. Moreover, an experiment and theoretical analysis carried out by Yamamoto and Nomoto (1975) on the impact point of two-dimensional and rectangular jets might show self-excited oscillation. However, they found that round jets are stable under all conditions.

Andreopoulos (1985) and Fric and Roshko (1994) discussed the detailed mixing behavior of a single jet and cross-flow. The mechanism of mixing on the turbulent jet boundary layer is becoming clearer through their research.

Although there has been much research on a single jet mixing in a cross-flow, there has been little research on opposing jets. Most research on opposing jets has studied the time-averaged concentration, and there has been very little research on the unsteady behavior of opposing jets. Also most research has not discussed the effect of instabilities at the collision point of mixing in the combustor. It is necessary to clarify unsteady phenomena to understand the opposing-jet mixing.

Lefebvre suggested specific parameters that affect the mixing phenomena in the combustor. The momentum flux ratio $J$ is the most emphasized parameter that strongly affects the trajectory and penetration of dilution air. The objective of this study is to examine how the momentum flux ratio $J$ affects the unsteady mixing of opposing jets. Unsteady computational fluid dynamics (CFD) are conducted for the mixing of opposing jets in a rectangular duct, which is the simplified shape of a gas turbine combustor, in a large eddy simulation (LES).
GEOMETRY
Figure 3 depicts the outline of the flow passage. As discussed in the previous section, the flow field consists of a rectangular duct and opposing jets. The left side of the duct is the cross-flow inlet, and the right side is open to the atmosphere. In this study, the entrance angle of the jets is fixed at 90 degrees to the duct. The entrance region length is \(X_a\) for the stability of calculation.

\[
\begin{align*}
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} &= -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \tau_{ij} - \frac{2}{3} \delta_{ij} \rho \bar{u}_i \bar{u}_j \right) \quad (1) \\
\tau_{ij} &= 2C_\mu \Delta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \quad (2)
\end{align*}
\]

The subgrid-scale (SGS) eddy viscosity was modeled using the standard Smagorinsky model as

\[
\tau_{ij} = 2C_\mu \Delta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \quad (3)
\]

where \(C_\mu\) is the Smagorinsky coefficient and \(\Delta\) is the length scale of the SGS turbulence. In this study, \(C_\mu\) was calculated with the dynamic SGS model (Lilly, 1992).

The minimum numerical mesh length was 0.2 mm, and the maximum length was 2 mm. There were 3 million grid points. The experiment was performed in our previous work (Nagao et al., 2012). According to the comparison between analysis and measurement, the LES was generally in quantitative agreement with the experiment.

Table 2 Calculated cases

<table>
<thead>
<tr>
<th>Momentum flux ratio</th>
<th>Jet velocity</th>
<th>Cross-flow velocity</th>
<th>Max. penetration (Eq. (6))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J)</td>
<td>(V_i) m/s</td>
<td>(V_m) m/s</td>
<td>(Y_{max}/H)</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>10</td>
<td>0.46</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>6.67</td>
<td>0.69</td>
</tr>
<tr>
<td>64</td>
<td>20</td>
<td>2.5</td>
<td>1.84</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

Unmixedness
Table 2 summarizes the conditions of the calculated cases. The physical properties of the cross-flow are the same as those of the jets. The inlet boundary condition of cross-flow has a uniform velocity distribution. Actually, this inlet condition is different from the actual combustor. However, a simplified boundary condition is used for the present numerical system to figure out the essence of its physics. Simulations were performed using air, where the average Reynolds number was set to \(2 \times 10^4\) according to the jet diameter and jet velocity.

Figure 4 shows the effect of the momentum flux ratio \(J\) on unmixedness \(U_i\). \(U_i\) is spatial unmixedness parameter (Vranos et al., 1991) which examined for the time-averaged results. In Fig.4 (a), \(U_i\) decreases with increasing \(J\) at the exit. However, the \(U_i\) decreases in the order of \(J = 4, 9, 64\) and \(J = 16\) in Fig.4 (b). Since the cross-flow velocity is variable, considering the mixing phenomena in terms of local residence time of axial direction \(T_r\) is reasonable (\(T_r = X / (Q / A)\)). In the case of \(J = 4\), \(U_i\) is high downstream; however, \(U_i\) is lower than the case of \(J = 9\) around the jet inlet (\(X/H < 1\)). A possible reason why \(U_i\) is low around the jet inlet is that the weak jet can easily be blown off and diffused by the cross-flow. Mixing phenomena are discussed in detail in the following section.

![Fig. 4 Effect of the momentum flux ratio on unmixedness](image-url)

(a) Horizontal axis: location
(b) Horizontal axis: residence time

Fig. 4 Effect of the momentum flux ratio on unmixedness
Scalar dissipation rate

Figure 5 presents (a) the instantaneous mass fraction of cross flow $f_j$, and (b) the scalar dissipation rate $\dot{\chi}$. The mass fraction is that of air entering from the left boundary where the total mass is the sum of the mass of air entering from the left boundary and the mass of both upper and lower air jet flows. Figure 6 indicates that the area average of scalar dissipation rate at each position of $Y/H$ in Fig. 5(b) from $X/H=1.0$ to $X/H=4.95$. The scalar dissipation rate is expressed in terms of the eddy diffusivity $D_j$ and the gradient of the resolved mixture fraction $\nabla f_j$ (Pitsch and Steiner, 2000) using the first-order model of Girimaji and Zhou (1996) written as

$$\ddot{\chi} = 2[D_z + D_j \nabla f_j^2].$$

where $D_z$ is the molecular diffusivity of the mixture fraction.

Although an increase in cross-flow velocity spreads the dilution zone on the downstream side, the scalar dissipation rate in the case of $J = 4$ seems to be low along the downstream walls (Fig 6). In contrast, distributions of the scalar dissipation rate in the other cases are more uniformly spread throughout the whole downstream area. There are differences in the distribution of the scalar dissipation rate $\ddot{\chi}$ according to the momentum flux ratio $J$.

Jet penetration and trajectory

Lefebvre and Ballal (2010) formulated jet trajectory and penetration in “Gas Turbine Combustion” from experimental results. Lefebvre empirically expressed the trajectory of a jet in cross-flow as

$$\frac{Y}{d_j} = 0.82 J^{0.5} (X/d_j)^{0.33}.$$  

where $d_j$ is the hole diameter, $Y$ is position of the jet center.

Figure 7 describes jet trajectories in uniform cross-flow according to Eq. (5). This equation has a good accuracy near the jet inlet. However, Eq. (5) indicates that $Y$ increases unlimitedly as $X$ increases. In fact, as a result that jet momentum is lost by the diffusion, the penetration length is limited. Hence the valid range of Eq. (5) is restricted by the maximum penetration distance $Y_{\text{max}}$ and the invalid range is described by the dashed-line in Fig. 7. Based on the Norster’s experiment, he found that the maximum penetration of a single jet into a circular duct is given by

$$Y_{\text{max}} = 1.15 d_j J^{0.5} \sin\theta.$$  

Table 2 shows the maximum penetration $Y_{\text{max}}$ is given by Eq. (6). The $Y_{\text{max}}$ is less than 0.5 in the case of $J = 4$, in which case the opposing jets do not collide with each other. Figure 7 shows that the case for $J = 4$ in which two jets are swept downstream by the cross-flow. These results indicate that the two jets have too little momentum to collide with each other. On the other hand, for larger values of $J$, $Y_{\text{max}}$ exceeds 0.5 and the jets are not swept strongly. Thus, higher unmixedness (Fig. 4) in the case of $J = 4$ will be due to lower jet momentum.
In general, the RMS of the mass fraction is high, where turbulence intensity is high. Hence, the RMS of the mass fraction might correlate with mixing. The RMS of the mass fraction in areas of jet collision is small in the cases of $J=9, 16, 64$. In Fig.8, these results suggest that the mixing intensity is strong in collision areas but weak in the downstream of cross-flow. In the case of $J=64$, the RMS of the mass fraction at the circumference of a radial jet is high, meanwhile, the K-shaped area also has high RMS values in the cases of $J=9$ and 16. This area shape is similar to the trajectory of the jet in Fig. 7, and it is thus suggested that the jet trajectory is correlated with the mixing phenomena. For further discussion, unsteady phenomena are considered in the next section.

**Mixing mechanism**

Figures 9–12 are the time sequence of the mass fraction on $Z=0$ and Figs. 13–16 are the time sequence of the iso-surface for a jet mass fraction of 0.4, where the jet profile is clearer than other value of jet mass fraction.

As discussed previously, in contrast to the case of $J=4$, two jets collide strongly in the cases of $J=9, 16, 64$. The sequential figures show that the jet fluctuates in all cases. The fluctuation seems to cause displacement of the collision point. The displacement strongly affects the direction of the collision point. If the jet has fluctuation, the direction of the collision plane also fluctuates. Accordingly, fluid is well mixed in the collision area. As clear examples, in the case of $J=64$ (Fig. 12), a radial jet is generated in the collision area. The radial jet fluctuates and stretches. In the case of $J=16$ (Fig. 11), low-concentration gas clumps are released in various directions from the point where two jets collide intermittently, and the high mixing rate (indicated in Fig. 8) associated with these clumps then spreads across a wide area.

What we are concerned here with is that no radial jet which flows upstream is observed in the case of $J=9$. It is considered that this is due to the high cross-flow velocity, even though the mixing is still strong. As illustrated in Fig. 14 ($J=9$), jets are dispersed and deflected in the area of collision, whereas the faster cross-flow case provides the weaker interaction of jets (Fig. 13 ($J=4$)), and the slower cross-flow cases give the stronger interaction of jets (Fig. 15 and 16). This result suggests that the mixing mechanism is different for $J=9$ and $J=64$ in Figs. 14 and 16. Note that in the case of $J=16$, the radial jet which flows upstream intermittently is observed at the time of 3.6 ms in Fig. 15(a). This indicate that the case of $J=16$ is the state between no radial jet flowing upstream case ($J=9$) and stable radial jet case ($J=64$).

Figure 17 shows the time series results of the position $Y_c$ of the maximum mole fraction at the position $X$, on the center plane.

Assuming that the position of the maximum mole fraction is the center of the jet trajectory, these results suggest that the jet trajectory fluctuates and is distributed in upper and downward directions.

The standard deviation of the position $Y_c$ fluctuation is shown in Fig. 18. The fluctuation of the jet increases in the order of $J=4, 9, 16$ and $J=64$. The large fluctuation of the jet trajectory results in wide dispersion of the jet component and good mixing. The quantitative relation between the dispersion of the jet component and the strength of the interference of the two jets result is obtained.

However, the standard deviation of the position $Y_c$ fluctuation in the case of $J=64$ is almost the same as that the case of $J=16$. Because in the case of $J=64$, the collision plane of two jets are stable, hence the fluctuation of $Y_c$ does not become too large considering mixing performance.

Furthermore, the fluctuation growth pattern on Fig. 18 does not increase monotonically in the case of $J=16$ and similar tendency in the case of $J=9$. In Fig. 8, the RMS of mass fraction monotonically decreases downstream in the case of $J=64$. Meanwhile, the high RMS region is distributed in K-shaped and because it non-monotonically decreases downstream in the case of $J=9, 16$.
their fluctuation growth patterns on Fig. 18 do not monotonically increase.

Finally, mixing mechanisms of calculated cases is represented in Fig. 19. The mixing mechanisms are differed depending on $J$. The mixing is not improved in the case $J$ that is too small for the collision of the two jets. The mechanism is similar to that for one jet in cross-flow. If jets collide with each other and $J$ is not large enough to generate a radial jet, each jet is dispersed or deflected by the opposite jet unsteadily. In the case that $J$ is larger, the mixing is improved by the radial jet, which fluctuates and stretches.
Fig. 13 Time sequence of the iso-surface for a jet mass fraction of 0.4 ($J = 4$)

Fig. 14 Time sequence of the iso-surface for a jet mass fraction of 0.4 ($J = 9$)

Fig. 15 Time sequence of the iso-surface for a jet mass fraction of 0.4 ($J = 16$)

Fig. 16 Time sequence of the iso-surface for a jet mass fraction of 0.4 ($J = 64$)
CONCLUSION

An experiment and simulations were performed to study the mixing of opposing jets in a rectangular duct. Thus, simulations were carried out to clarify the effect of the momentum flux ratio \( J \). In this study, the values of \( J \) were set at 4, 9, 16 and 64. The result for unmixing was especially high in the case of \( J = 4 \). The reason is that the mixing intensity was not enhanced by the collision of opposing jets since the penetration is weak in the case of \( J = 4 \). In summary, mixing mechanisms of calculated cases is represented in Fig. 19. The mixing mechanisms differ depending on \( J \). The mixing is not improved in the case that \( J \) is too small for the collision of the two jets. The mechanism is similar to that for single jet in cross-flow. If jets collide with each other and \( J \) is not large enough to generate a radial jet, each jet is dispersed or deflected by the opposite jet unsteadily. The large fluctuation of the jet trajectory results in wide dispersion of the jet component and good mixing. In the case that \( J \) is larger, the mixing is improved by a radial jet that fluctuates and stretches. Note that Lefebvre suggested that the \( D/H \) and \( W/H \) ratios are important conditions. Further studies are needed to verify the mixing mechanism of opposing jets.

NOMENCLATURE

\[ A = \text{cross section area of the duct} \]
\[ C_{\text{avg}} = \text{fully mixed mass fraction} = \frac{w_m}{(w_j + w_m)} \]
\[ C_{\text{rms}} = \text{root-mean-square of the mass fraction} = \left( \frac{1}{n} \sum_{i=1}^{n} (C_i - C_{\text{avg}})^2 \right)^{1/2} \] (Vranos et al., 1991)
\[ C_i = \text{average concentration at a point} \]
\[ D = \text{diameter of the oriﬁce} \]
\[ f_c = \text{Mass fraction of cross-ﬂow} = f_c + f_m + f_j = 1 \]
\[ f_m = \text{Mass fraction of upper jet} \]
\[ f_l = \text{Mass fraction of lower jet} \]
\[ H = \text{duct height in the injection plane} \]
\[ J = \text{jet-to-cross-ﬂow momentum flux ratio} = \left( \frac{\rho_j V_j^2}{\rho_m V_m^2} \right) \]
\[ n = \text{number of acquisition points in each cross section} \]
\[ Q = \text{total volume ﬂow rate} \]
\[ T_r = \text{local residence time of the fluid} = \frac{X}{(Q / A)} \]
\[ T_{all} = \text{overall passing time through the region} = \left( \frac{L - X_c}{Q / A} \right) \]
\[ U_j = \text{spatial unmixedness parameter} = C_{\text{rms}} / C_{\text{avg}} \]
\[ V_j = \text{jet velocity} \]
\[ V_m = \text{cross-ﬂow velocity} \]
\[ W = \text{spacing between sidewalls} \]
\[ w_j = \text{mass ﬂow rate of jets} \]
\[ w_{mr} = \text{mass ﬂow rate of the cross-ﬂow} \]
\[ w_j/w_{mr} = \text{jet-to-cross-ﬂow mass ﬂow ratio} \]
\[ X, Y, Z = \text{position of X, Y, Z-axis} \]
\[ X_c = \text{position of arbitrary point on Z} = 0 \]
\[ Y_c = \text{position of the maximum mole fraction on X_c} \]
\[ \rho_j = \text{jet density} \]
\[ \rho_m = \text{cross-ﬂow density} \]

References


