Numerical and experimental studies on self sustained thermo-acoustic combustion instabilities of an experimental rig for full-scale industrial burners

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ABSTRACT
In this work a three-dimensional finite element (FEM) code is used to perform the stability analysis of an experimental rig designed and operated in order to study the propensity of full-scale industrial burners to thermo-acoustic combustion instabilities. The Burner Transfer Matrix (BTM) approach is used to characterize the influence of the burner. An experimental transfer matrix is used and compared with an analytical model. In the combustion chamber, the influence of the three—dimensional spatial distribution of the thermodynamics quantities in presence of the flame is considered. Reynolds Averaged Navier-Stokes (RANS) computational fluid dynamics (CFD) simulations are used to compute the base flow. A linear stability analysis of the entire system is performed considering the coupling between pressure oscillations ($p'$) and heat release fluctuations ($q'_f$) with a distributed n-τ linear Flame Transfer Function (FTF). A three-dimensional spatial distribution of time delay ($\tau$) is reconstructed assuming the delay due to convection as the predominant effect. Under these assumptions a good agreement between numerical and experimental results is found both in terms of thermo-acoustic resonance frequencies and mode shape of the resonant modes for different setups of the analyzed system.

INTRODUCTION
Modern gas turbines for power generation equipped with lean premixed dry low NOx combustion systems suffer the problem of combustion instabilities. Combustion instabilities are characterized by self-sustained pressure oscillations, which arise from the coupling between unsteady heat source produced by the flame and the acoustic waves of the system. This feedback loop behavior will result in a deterioration of combustion efficiency, an increase of pollutant emissions (NOx in particular) and a lowering of the power production. Furthermore, the pressure and velocity oscillations develop self-sustained vibrations which may cause fatigue cycles on system elements and blow-off or flashback phenomena of the flame.

In general, the thermo-acoustic oscillations are associated with one of the natural pure acoustic modes of the system. These include, for example, bulk, axial, and transverse modes [1]. In real gas turbine systems, combustion instabilities coupled by azimuthal modes are the most susceptible to be triggered due to the annular configuration of the combustion chambers. In order to study the propensity of a given burner to this type of instabilities an annular rig should be used. There are few laboratory scale rigs where these instabilities could be investigated [2, 3]. However, full-scale burners cannot be tested in such devices and real machines have to be used, with a substantial increase of the experimental costs. A more practical solution is to design a longitudinal setup to study the combustion dynamics of only one single burner excited at the same frequencies observed in the annular combustor.

From a numerical point of view, over the years different approaches have been developed in order to model a thermo-acoustic system and to define a method able to predict the onset of thermo-acoustic instabilities. Computational fluid dynamics (CFD) simulations can theoretically detect all the main effects of the phenomenon. Particularly, Large-Eddy Simulation (LES) codes are proposed in order to investigate combustion instability and matching pressure oscillations with turbulent combustion phenomena [4–6]. Large numerical resources are, however, required. The less computationally demanding models are the low-order models. Low-order models are based on the idea of modeling thermo-acoustic system as a network of acoustic elements. Generally, acoustic networks represent a very strengthened methodology, providing fast answers. These models are very attractive because they provide non-trivial results and they are very helpful to understand the instability mechanisms, in particular, the phenomena correlated to related to the mean flow, such as entropy waves [7] and vorticity [8]. However, low-order models can become inexact when complex elements are involved.

The present investigation reports a numerical analysis on the longitudinal combustor developed by Ansaldo Energia and CCA (Centro Combustion e Ambiente) to test the propensity to instabilities of their full-scale burners. The rig is examined with a hybrid technique based on a finite element method approach [7, 9]. With this approach, the inhomogeneous wave equation can be solved for three-dimensional geometries [7, 9]. The analysis is performed in the frequency domain considering the heat release fluctuation as a source term into the acoustic wave equation. At first a purely acoustic modal analysis is carried out, i.e., no heat release fluctuation is considered. Two different burner transfer matrices, one analytical and one experimental, are considered and compared in this section evaluating frequency and damping rate of the pure acoustic modes of the system. Subsequently, in the flame zone heat release fluctuations are coupled to velocity oscillations occurring at the injection point with a distributed n-τ linear flame transfer function [1, 10] derived considering a numerical distribution of time delays $\tau(\ell)$. A linear stability analysis is performed highlighting the influence of the variation of the length of the two main volumes of the system: the plenum and the combustion chamber. In the last part of this work, the numerical scattering matrix of the system is computed in order to evaluate the amplification factor of the entire system and highlight the damping role of the plenum.
THE EXPERIMENTAL CONFIGURATION
The longitudinal rig analyzed in this work has been thoroughly used to study the dynamics full-scale burners [11–14]. It comprises of two cylindrical chambers, the plenum and the combustion chamber, coupled by the burner. Both chambers are equipped with a movable disk, whose longitudinal position can be continuously varied during the tests to induce instabilities at different frequencies. The length of the plenum can be reduced of a ∆L = 500 mm, whereas the combustion chamber has a maximum excursion of ∆Lc = 3000 mm. In order to assure an adiabatic condition, a layer of refractory material covers the combustion chamber. As it is possible to notice in the Fig. 1(a) the air supply system is not co-axial with the plenum, however the length of the chamber is sufficient to homogenize the flux and avoid non-uniformity at the inlet of the diagonal swirler of the burner. Air inlet temperature, air mass flow rate and air-to-fuel ratio are continuously monitored and controlled. In order to provide an acoustic decoupling upstream of the plenum, a fine grid is interposed between the plenum and the air supply conduit. For the same reason, the end of the combustion chamber consists of a multi-holed disk characterized by a porosity σ = 7.4% defined as the flow passage area over the section of the combustion chamber. Measurements of the acoustic pressure, both in the plenum volume and in the combustion chamber, are conducted and the resonant modes are reconstructed by means of the Multi-Microphone-Method (MMM) [15].

The full-scale gas turbine burner tested in this work is shown in Fig. 1(b). Air is injected through two different coaxial swirlers (diagonal and axial) with the main air mass flow passing through the diagonal passage. Fuel is injected without being pre-heated through small holes in the vanes of the diagonal passage. At the exit of the burner, a cylindrical volume called Cylindrical Burner Outlet (CBO) is located. In all configurations studied in this work, the burner is operated only in lean premixed condition characterized by a very lean equivalent ratio of φ = 0.51 slightly above the flammability limit [16].

BASELINE FLOW THERMODYNAMICS QUANTITIES
Reynolds Average Navier Stokes (RANS) simulations are performed in order to calculate the three-dimensional fields of the thermodynamics quantities of the baseline flow. The six species-three reactions 2sCM2 combustion scheme has been used to simulate the combustion process [4, 13]. Fig. 2 reports the contour plots of the field of temperature (Fig. 2(a)) and reaction rate (Fig. 2(b)) of the first-step reaction (the CH4 oxidation) taken in a diametrical plane of the CBO-combustion chamber domain. The values of temperature is reported normalized with respect to the corresponding value of the inlet air, which is assumed constant in the plenum. The reaction rate is normalized with respect to the maximum value. The longitudinal and transverse coordinates are normalized respectively with the length of the combustion chamber (Lc) and the combustion chamber radius (R). From Fig. 2(b), it is possible to notice that the first part of the flame front is located within the CBO that is included in the computational domain. Further details of these RANS simulations and a comparison between results from LES simulations can be found in [13].

![Fig.1 (a) Experimental Configuration; (b) full-scale swirled burner details [6]](image)

![Fig.2 Contour plots of the baseline flow quantities: (a) normalized temperature (T/T_inh); (b) normalized reaction rate (∂RR/∂RR_max)](image)

THERMO-ACOUSTIC MATHEMATICAL MODEL
The derivation of the mathematical model used for thermo-acoustic studies will be briefly discussed in this section. The complete formulation can be found in several works [17, 18]. In presence of small perturbations, a generic flow variable (a) can be decompose as the sum of two terms

\[ a = \bar{a} + a' \]  

where the over-bar indicates a mean quantity and the prime indicates a perturbation over the time. An approximate linear wave equation for pressure perturbations \( p'(x,t) \) for reacting flows may be derived applying the decomposition of Eq. (1) to the Navier-Stokes equations with the auxiliary hypotheses of treating the fluid as an ideal gas and neglecting the viscous losses and heat conduction

\[ \frac{1}{\gamma} \nabla^2 p' + \nabla q' \nabla - \frac{\gamma - 1}{\gamma} \frac{\partial q'}{\partial t} = 0 \]  

where \( q' \) represents the fluctuation of the heat release rate per unit volume, \( p' \) is the mean density, \( t \) and \( c \) are, respectively, time and sound velocity. Eq. (2) relies on the so called zero-Mach-number-approximation [19], which state that the mean velocity is very small if compared to the speed of sound, so is considered null. On the applicability of this assumption several studies have been performed [20, 21]. The main reason is that Eq. (3) does not support entropy waves, so the thermo-acoustic instabilities due to entropy
spot being accelerated at the end of the combustion chamber is not accounted for and mixed modes cannot be captured.

Generally, the thermo-acoustic analysis is carried out in the frequency domain. In the harmonic analysis, a generic fluctuating quantity is written as $q' = \Re\{\hat{q}\exp(i\omega t)\}$, where $\hat{q}$ is a complex quantity, $i$ the imaginary unit and $\omega$ is the complex angular frequency. Introducing the harmonic fluctuations of pressure oscillations and heat release oscillations and considering a spatial variation of the base flow thermodynamic variables, Eq. (2) becomes

$$
\frac{\lambda^2}{\rho(x)} \vec{p}(x) - \vec{p}(x) \nabla \cdot \left( \frac{1}{\rho(x)} \nabla \vec{p}(x) \right) = -\frac{\gamma - 1}{\rho(x)} \lambda \hat{q}(x), \quad (3)
$$

where $\lambda = -i\omega$. Equation (3) is referred to as the non-homogeneous Helmholtz equation.

**Flame transfer function**

In order to close the problem, a relation to correlate the unsteady heat release fluctuations with the pressure waves is needed. Following [1], a time delay $n-\tau$ model is assumed

$$
\frac{q'(x,t)}{\hat{q}(x)} = -n(x) \left( \frac{u(i(\tau - x))}{\tau} \right), \quad (4)
$$

where $n(x)$ is the acoustic-combustion interaction index that controls the amplitude of the flame response and $\tau(x)$ is the time delay between the acoustic perturbation and the flame response. The subscript $i$ denotes the location of the reference point for the velocity fluctuations. In this work, this point is taken at the injection location inside the burner. Mean heat release rate is defined as $\hat{q}(x) = \Re\{q\} - h_f$, where $\Re\{q\}$ represents the spatial distribution of the reaction rate $\text{kmol/m}^3s$ showed in Fig. 2(b), $h_f$ is the Lower Heating Value of the fuel. In this analysis, pure CH$_4$ is considered, i.e., $h_f = 50 \text{MJ/kg}$. Eq. (4) has been already proved to be suitable to account the thermo-acoustic instability due to fuel–oxidizer mixture perturbations especially if the analysis regards the low frequency region [22]. At high frequency a flame saturation model should be included [23]. In other cases [24, 25], thermo-acoustic instability may occurs due to a direct interaction between the flame and the acoustic wave, i.e., changes in the flame area or flame-vortex interactions. The $n-\tau$ model and the Helmholtz solver approach are not suitable to study these phenomena, which require LES simulations.

In the frequency domain, the distributed Flame Transfer Function ($\mathcal{F}$) is derived from Eq. (4)

$$
\hat{q}(x) = -n(\omega, x) \exp(-i\omega \tau(\omega, x)) \hat{u}_i, \quad (5)
$$

with both the interaction coefficient $n$ and the time delay $\tau$ which are function of the frequency. In this study, $n$ and $\tau$ are assumed constant with the frequency. In particular, $n$ is assumed equal to unity, while a spatial three-dimensional distribution of time delay is used in the simulations. This distribution is obtained following the procedure proposed in [26]: only the convective time [22] required by a particle injected at the reference section in the burner to reach the flame front is assumed as time delay. All other contributions defined in Ref. [22] are neglected. Figure 3 shows a contour plot of the normalized time delays on the flame front.

**Numerical model**

Thermo-acoustic simulations are performed by means of a three-dimensional finite elements (FEM 3D) Helmholtz solver approach [9, 28]. The computational domain used in this work is shown in Fig. 4. The plenum consists of a cylindrical volume partially occupied by the burner’s body. All the auxiliary systems located upstream of the inlet boundary condition (highlighted in Fig. 4) are not directly modeled. The plenum inlet is considered to behave like a rigid wall ($n' = 0$) to achieve a perfect acoustic decoupling condition.

A complex value of acoustic impedance ($\hat{z}_d$) is imposed as outlet boundary condition to take into account the acoustic influence of the elements downstream of the perforated orifice. The value of $\hat{z}_d$ is calculated as [29, 30]

$$
\hat{z}_d = \frac{\hat{p}}{\hat{p}_w} = \mathcal{F}\frac{1 + r}{1 - r}, \quad (6)
$$

where $r$ is the reflection coefficient measured during tests. In Fig. 5, magnitude and phase of the experimental value of the reflection coefficient is reported. As it is possible to see the mobile holed disk located at the end of the combustion chamber is not able to provide a perfect decoupling with the exhaust systems.

The computational domain is discretized with an unstructured mesh of tetrahedral elements. Figure 6 shows the mesh refinement performed in the CBO domain and in the first part of the combustion chamber in order to correctly simulate the area interested by the combustion process.

**The Burner modeling**

For the modeling of the burner, the transfer matrix approach (TM) has been used [26, 31]. With respect to a direct FEM modeling, the transfer matrix allows to account the influence of acoustic energy dissipation due to fluid dynamic and viscous losses. The
The applicability of this modeling approach to compact elements characterized by complex geometries, such as the full-scale industrial burner considered in this work, has already been proved in Laera et al. [12] where the stability of the longitudinal combustor was investigated using the following analytical transfer matrix proposed by Fanaca and Alemela [32]

\[
\begin{bmatrix}
\bar{p}_d \\
\bar{u}_d
\end{bmatrix} = \begin{bmatrix}
1 & M_u - \alpha \Delta M_d (1 + \zeta) - j keff \\
\alpha \Delta M_u & \alpha + M_d keff
\end{bmatrix} \begin{bmatrix}
\bar{p}_u \\
\bar{u}_u
\end{bmatrix},
\]

(7)

where subscripts \( u \) and \( d \) respectively refer to the section upstream and downstream of the element, \( \bar{p} \) is the mean density of the baseline flow, \( \bar{u} \) the mean speed of sound, \( k \) the wave number, \( M \) the Mach number of the base flow, \( \alpha = A_u/A_d \) is the area ratio, \( \zeta \) a pressure loss coefficient, \( keff \) is a measure of the accelerated mass in the compact element. Complete details on these coefficients and their analytic derivation can be found in [32].

In this work, the transfer matrix of Eq. (7) is substituted by an experimental transfer matrix calculated on the real full-scale burner. The experimental transfer matrix of the burner. In Fig. 7 the scheme of the test rig used for the calculation of the transfer matrix of the burner is reported. The end of the plenum is replaced by a “Helmholtz siren” that it’s able to force the system with a perfect sinusoidal pressure signals elaborating part of the inlet mass flow rate. A standard Multi-Microphone-Method approach has been used to reconstruct the pressure signal both in the plenum and in the combustion chamber. For these measurements, the rig has been equipped with three microphones in the plenum and four in the combustion chamber. In order to have the two sets of independent experimental data required to calculate the four coefficients of the matrix the “two load method” approach has been used [33]. A different outlet condition is created operating with the length of the combustion chamber (indicated as “State 1” and “State 2” in Fig. 7). The magnitude and phase of each element of the experimental transfer function are reported respectively in Fig. 8(a) and Fig. 8(b). The frequency on the x-axis is normalized and reported in terms of the Helmholtz number (He), which is defined as \( He = \omega D/c \) with \( D \) the mean diameter of the diagonal swirl. The phase values are normalized with respect to \( \pi \). A comparison with the analytic model of Eq. (7) is reported for each element of the transfer function (hashed black lines). As it is possible to notice the transfer matrix model of Eq. (7) is not able to completely describe the acoustic behavior of the full-scale industrial burner analyzed. Differences appear in the magnitude of coefficients \( T_{11} \), \( T_{12} \) and \( T_{22} \), where the analytical values seem to tend to the mean values, and in the phase of elements \( T_{11} \) and \( T_{12} \). A good agreement is obtained for the magnitude of the element \( T_{21} \) and the phase of the elements \( T_{11} \) and \( T_{22} \). Figure 9 shows the implementation of the transfer matrix in the computational domain.
RESULTS

Different operative conditions characterized by different lengths of the plenum and the combustion chamber are examined. In all cases, the thermo-acoustic analysis concerns only the modes of the combustion chamber within the azimuthal cut-off frequency. Results considering the theoretical transfer matrix of Eq. (7) and the experimental matrix are at first compared in order to underline the differences. In the frequency domain, two different types of studies are performed: at first a pure acoustic analysis (“passive flame”) is carried out, subsequently, the heat release fluctuation is considered (“active flame”). Results are express in terms of resonant frequency and growth rate of the modes. Due to confidentiality reasons, frequencies and growth rates are reported normalized with respect to the frequency of the first resonant mode, whereas the length of plenum and combustion chamber are normalized with respect to the maximum value of the length of the combustion chamber. Results of these studies are presented hereafter.

“Passive flame” Simulations

In this analysis, no heat release rate fluctuation is considered, meaning that the RHS of Eq. (3) is null everywhere in the computational domain. Under such condition, the simulation is purely acoustic and the results are the acoustic modes of the system around which the thermo-acoustic oscillations occurs [1]. For this analysis the length of the combustion chamber is set at \( l_{cc}/l_{cc,max} = 0.84 \), whereas the plenum is considered at its maximum extension, i.e., \( l_p/l_{cc,max} = 0.64 \). Tab. 1 reports the normalized resonant frequencies and growth rates for the two different approaches used to model the burner. Although the two parts of the rig are connected to each other by means of the burner, it is possible to identify some regions where the amplitude of a given mode is predominant and the corresponding mode is more sensitive to acoustic or geometric changes of that specific region. In Tab. 1 the main region in which acoustic waves propagate, combustion chamber (CC) or plenum (P), is indicated for each frequency value. Focusing the attention on the modes of the combustion chamber (reported in bold) it is possible to observe that the different modeling approach has no influence on the frequencies of the resonant modes. All modes present a negative growth rate \( \alpha \) \(^1\) due to the fact that no heat release rate fluctuation is considered, however, it can still be used to estimate the acoustic damping of the mode. Comparing the growth rates, it is possible to observe that the analytic model of Eq. (7) is not able to reproduce the acoustic losses of the analyzed burner. At low frequencies (modes II and IV of Tab. 1) the model introduces a not sufficient level of damping, on the contrary the high frequencies are predicted over-damped (mode VI of Tab. 1). For what concerns the modes of the plenum, a slight variation of the resonant frequencies is registered. This result can be explained by a different plenum–combustion chamber coupling level imposed by the transfer function [34]. However, for both modeling approaches the level of damping computed for the plenum modes is higher than the one predicted for the modes of the combustion chamber, meaning that for the analyzed system instabilities will be coupled to a chamber mode. The wave shape of the three resonant modes of the combustion chamber for the configuration with the experimental transfer function are reported in Fig. 10 in terms of normalized pressure fluctuations.

“Active flame” simulations

In this section the heat release is considered in the simulations and a linear stability analysis is performed solving the complex eigenvalue problem of Eq. (3) with the distributed flame transfer function of Eq. (5). In these analyses, only the experimental burner transfer matrix will be used in order to include in the model the correct level of damping.

\(^1\)In this case is better to talk about of “dissipation rate \( \alpha \)”

<table>
<thead>
<tr>
<th>Modes</th>
<th>( f_n )</th>
<th>( \alpha )</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.02</td>
<td>-1.5E-2</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>1.96</td>
<td>-3.5E-2</td>
<td>-4.1E-3</td>
</tr>
<tr>
<td>III</td>
<td>2.03</td>
<td>-1.3E-2</td>
<td>1.99</td>
</tr>
<tr>
<td>IV</td>
<td>2.88</td>
<td>-2.9E-2</td>
<td>2.92</td>
</tr>
<tr>
<td>V</td>
<td>3.04</td>
<td>-9.3E-3</td>
<td>3.04</td>
</tr>
</tbody>
</table>

Table 1 Passive Flame Simulation: Normalized resonant frequency, growth rate and type of the mode. Burner Transfer Matrix: (A) Experimental; (B) Analytical. (CC stands for combustion chamber and P for plenum)

The influence of the length of the combustion chamber

At first, the influence of the length of the combustion chamber is analyzed. The operative conditions of the burner are kept constant for all analyzed configurations. In Fig. 11 is shown the experimental trend of the rms of the acoustic pressure registered by two microphones placed in the combustion chamber. These results refer to two different configurations of the rig that are characterized by two lengths of the combustion chamber: \( l_{cc}/l_{cc,max} = 0.84 \) (Fig. 11(a)) and \( l_{cc}/l_{cc,max} = 0.5 \) (Fig. 11(c)). It is possible to notice that in both cases a limit cycle condition is reached. The pressure is reported normalized with respect to the maximum value, the normalization factor for the frequency is the same used for the “passive flame” simulations. The normalized frequency of the two unstable modes is shown respectively in Fig. 11(b) and Fig. 11(d) where the spectrum of the dynamic signals of all four microphones placed in the combustion chamber is shown. For the other values of the combustion chamber length no instabilities have been observed.

For the numerical analysis, in Fig. 12 is reported the linear stability curve of the system colored with respect to the value of the combustion chamber. The length of the combustion chamber is varied from \( l_{cc}/l_{cc,max} = 0.84 \) to \( l_{cc}/l_{cc,max} = 0.5 \). Each point of the curve corresponds to a stability analysis of the system, in terms of frequency \( f_n \) and growth rate \( \alpha \), performed for a specific value of length of the combustion chamber. A good agreement with the experimental results is observed. In particular, it is possible to notice that in the two configurations where experimentally instabilities are registered, i.e., \( l_{cc}/l_{cc,max} = 0.84 \) and \( l_{cc}/l_{cc,max} = 0.5 \), two unstable modes (\( \alpha > 0 \)) have been numerically found (indicated with a red star in Fig.12). The frequency of these modes is very close to \( 2l_{cc}/l_{cc,max} = 0.5 \) is the minimum physically extension of the combustion chamber.

\[
l_{cc}/l_{cc,max} = 0.5
\]
Fig. 11 Experimental analysis: the influence of the length of the combustion chamber. Dynamic pressure in the combustion chamber (a) and spectrum (b) in the rig configuration with the length of the combustion chamber equal to \( l_{cc}/l_{cc,max} = 0.84 \). Dynamic pressure in the combustion chamber (c) and spectrum (d) for the rig configuration with the length of the combustion chamber equal to \( l_{cc}/l_{cc,max} = 0.5 \).

As registered during experiments, for the other configurations, the system results stable, i.e., \( \alpha < 0 \), except that in proximity to the unstable configuration where small variations of the length of the combustion chamber are not able to fully decouple the feedback between the heat release fluctuation and the pressure fluctuations as already shown in [12].

Fig. 12 Numerical analysis: the influence of the length of the combustion chamber. Stability trajectory of the test rig colored with respect to the length of the combustion chamber. By means of the Multi-Microphones-Method it is possible to reconstruct the experimental wave shape of the unstable modes both in the plenum and in the combustion chamber. These results are reported in Fig. 13 for the \( l_{cc}/l_{cc,max} = 0.84 \) configuration and in Fig. 14 for the case with \( l_{cc}/l_{cc,max} = 0.5 \). In both diagrams, the acoustic pressure is normalized with respect to the maximum value. The axial dimension both in the plenum and in the combustion chamber is normalized with respect to the length of the combustion chamber. The origin for the axial reference system is considered at the beginning of the combustion chamber. Considering the position of the maximum of the normalized pressure and the decoupling level imposed by the burner, it is possible to conclude that in both configurations the thermo-acoustic coupling between the pressure fluctuations and the heat release rate is around the first longitudinal mode of the combustion chamber. In the same figures, a comparison between the numerical and experimental wave shape of the two resonant modes both in the plenum and in the combustion chamber is also shown. It is possible to notice that in the combustion chamber the model is able to reproduce perfectly the wave shape of the mode in both cases. On the contrary, in the plenum the agreement is not as good as in the combustion chamber. In particular for the configuration with \( l_{cc}/l_{cc,max} = 0.84 \) (Fig. 13(a)) an error on the position of the minimum pressure occurs. The difference is about 10%. In the case with \( l_{cc}/l_{cc,max} = 0.5 \) (Fig. 14(a)) a differences between the numerical and experimental mode occurs only in proximity of the second pressure nodes\(^3\). In both cases, these discrepancies occur in the second half of the plenum volume and can be explained due to a non–perfect modeling of the inlet boundary condition of the plenum.

Fig. 13 Normalized absolute pressure of the unstable mode (\( f_n = 0.91, \alpha = 0.33E - 2 \)) in the configuration with the length of the combustion chamber equal to \( l_{cc}/l_{cc,max} = 0.84 \). Wave shape reconstruction in the plenum (a) and in the combustion chamber (b). Comparison with experimental measurements.

\(^3\)Starting from left to right following the orientation of the x-axis.

The influence of the length of the plenum

In this section, the effects on the instabilities of the variation of the plenum geometry is investigated. The length of the combustion chamber is kept constant when the length of the plenum is changed. This analysis is performed for the two unstable configuration described in the previous section.

In Fig. 15(a) the rms of the dynamic pressure registered in the combustion chamber for two different pressure sensor under an un-
Fig. 14 Normalized absolute pressure of the unstable mode \((f_n = 1.56, \alpha = 4.1E -2)\) in the configuration with the length of the combustion chamber equal to \(l_{cc}/l_{cc,max} = 0.5\). Wave shape reconstruction in the plenum (a) and in the combustion chamber (b). Comparison with experimental measurements.

stable condition is reported. On the same diagram the length of the plenum is plotted (dashed line), reported in terms of \(\Delta L_p = l_{p,max} - l_p\). Experimentally has been found that it is possible to notice that starting from a limit cycle condition, a reduction of the length of the plenum (negative \(\Delta L_p\)) induces a damping of the amplitude of the pressure oscillations. When the length of the plenum is restored at its maximum value, the oscillations amplitude returns at its original level. Figure 15(b) and Fig. 15(c) show the spectrum of the pressure signal taken in the configuration of \(\Delta L_p = 0\) mm and \(\Delta L_p = -500\) mm. It is possible to notice that the frequency of the resonant mode is unchanged.

Numerically, to investigate this behavior a linear stability analysis is performed. Figure 16 reports the linear stability curve of the system colored with respect to the values of plenum lengths. The analysis is performed for the two unstable configurations with \(l_{cc}/l_{cc,max} = 0.84\) and \(l_{cc}/l_{cc,max} = 0.5\) (Fig. 16(a) and Fig. 16(b) respectively). It is possible to notice that in both cases reducing the length of the plenum results in a reduction of the growth rate \((\alpha)\) of the resonant modes, whereas the frequency remains constant. For the case with \(l_{cc}/l_{cc,max} = 0.65\) a shift between a unstable to a stable condition is registered. Qualitatively this is the same result that has been found experimentally considering that for a given level of acoustic energy losses, a lower growth rate in the linear stability analysis will correspond to a lower amplitude of oscillations at limit cycles. The aim of this analysis is not to perfectly reproduce the experimental results, objective that would have required an experimental nonlinear Flame Describing Function [14, 28, 35] and a perfect modeling of all acoustic dissipation sources, but to reproduce the trend and understand causes. In this contest, the shift between a stable to an unstable condition is useful because the differences between the two conditions are highlighted and easier to interpret.
Figure 17 shows the wave shape of the analyzed mode in the combustion chamber and in the plenum for different lengths of plenum. Since this is the results of an eigenvalue analysis, the acoustic pressure is reported normalized with respect to the maximum value, so no information on the absolute value of the amplitude of pressure oscillations is given. The longitudinal coordinate in the combustion chamber and in the plenum is normalized with respect to the length of the combustion chamber (fixed in this analysis). In the combustion chamber (Fig. 17(b)) it is possible to notice that the position of the pressure node remains constant with the variation of the plenum length. This is an expected result since the analyzed mode is a mode of the combustion chamber and no changes have been made at the combustion chamber acoustic volume during this study. Differently, in the plenum (Fig. 17(a)) a variation of the wave shapes is registered changing the length. When the length is reduced, it is possible to notice a shift of the position of the pressure nodes towards the burner inlet section. An increase of the relative pressure level, especially in the upstream section of the burner, is also registered.

Energy analysis of the system. In order to compute the acoustic losses, an energy analysis of the entire system is performed [36]. Given a multi-port acoustic element, such as the computational domain considered in this work, the scattering matrix

\[
S(\omega) = \text{mathematical operator that relates the characteristic wave amplitudes incident on the multi-port element, i.e., } f_u \text{ and } g_d, \text{ with the characteristic wave amplitudes emitted, i.e., } f_d \text{ and } g_u
\]

\[
\left( \begin{array}{c} f_u \\ g_u \end{array} \right) = S \left( \begin{array}{c} f_d \\ g_d \end{array} \right),
\]

where the coefficients of the scattering matrix can be identified as transmission and reflection coefficients \( t \) and \( r \) of the waves \( f_u \) and \( g_d \)

\[
S = \left( \begin{array}{cc} r_u & t_u \\ r_d & t_d \end{array} \right).
\]

Under the zero-Mach-number assumption, the acoustic power \( P \) (flux of acoustic energy) in a cross sectional area \( A \) can be defined as [17]

\[
P = \frac{1}{2} \rho c |f|^2 A = \frac{1}{2} \rho c \left( |f|^2 - |g|^2 \right) A \ \ [W].
\]

For the wave \( f_u \) traveling towards the multi-port system and for the wave \( g_d \) incident on the multi-port system an amplification factor of the acoustic energy can be defined as \( \Pi = P_{in}/P_{out} \), where \( P_{in} \) is the acoustic energy of the incoming waves. Following Gentemann and Polijcke [37], if an anechoic boundary condition is assumed, a correlation of the scattering matrix coefficients and the amplification factor is founded. The anechoic boundary condition is not a limitation because, under the limit of a linear analysis, the frequency response of the system is not influenced by the nature of the boundary conditions, so the coefficients of the scattering matrix are independent from the chosen boundary conditions. For example, if \( g_d = 0 \)

\[
\Pi_{fu} = \frac{\rho c A |f|^2_1 + \rho c A |g|^2_2}{\rho c A |f|^2_1} = \frac{\rho c A |f|^2_1}{\rho c A |f|^2_1} \left( |S_{11}|^2 + |S_{21}|^2 \right),
\]

and vice versa if \( f_u = 0 \)

\[
\Pi_{fd} = \frac{1}{2} \rho c A |f|^2_2 + \frac{1}{2} \rho c A |f|^2_1 |S_{12}|^2.
\]

\( \Pi \) lower then unit means that a damping of acoustic energy occurs in the system. If instability occurs, \( \Pi \) needs to be greater than one. If the amplification factor is equal to the unity for all the frequencies, no dissipation or amplification of the acoustic energy occurs in the system.

Similar conclusions can be reached observing the magnitude of the coefficients of the scattering matrix: if instability occurs, at least one of the four coefficients of the matrix would be greater than the unity. Figure 18 shows magnitude and phase of the four coefficients of the scattering matrix calculated in the configuration with \( \Delta L_p = 0 \) mm under the condition of passive flame, i.e., no heat release rate fluctuations are considered in the simulations. The matrix of the entire system is computed by means of the “two-source location” technique [33]. The analysis is performed in a range of frequencies around the frequency of the first unstable mode of the combustion chamber. It is possible to notice that, the magnitude of all four coefficients \( |S_{ij}| \) is below unity as expected because no flame is considered and the burner is a dissipative element.

In Fig. 19 the amplification factor for the incident wave \( f_u \) for different lengths of the plenum is showed. Equation (11) is used for the calculations. In all the configurations \( \Pi_{fu} \) is less than unity, as expected since no heat release fluctuations are considered in this analysis. However, it is possible to notice that reducing the length of the plenum a decrease of the amplification factor is registered. In other words, the reduction of the plenum length produces an increase of the magnitude of the dissipation of acoustic energy in the burner.
CONCLUSIONS

In this work, a code based on the Finite Element Method able to predict the onset of thermo-acoustic instabilities is applied to an experimental test rig in order to verify the ability of the method to provide a description of the phenomenon. The experimental test rig is designed to evaluate the propensity to thermo-acoustic instabilities of full-scale Ansaldo Energia burners used in gas turbine systems for energy production.

Reynolds Average Navier Stokes (RANS) simulations have been performed in order to calculate the baseline flow thermodynamics quantities. A distributed n–τ flame model with a three–dimensional fields of time delays has been considered in the simulations. The Burner Transfer Matrix (BTM) approach has been used to simulate the burner. Two different burner transfer matrix, one analytical and one experimental, are compared in terms of frequency and dissipation rate of the acoustic resonant modes. The comparison of the results has highlighted that the analytic model is not able to reproduce the acoustic behavior of the analyzed full-scale swirled burner.

In the frequency domain, a linear stability analysis is performed and the influence of the variation of the length of the two volume, the plenum and the combustion chamber, is studied. For a fixed value of the length of the plenum, the rig was found unstable only for two values of the combustion chamber, i.e., \( l_{cc}/l_{cc,max} = 0.84 \) and \( l_{cc}/l_{cc,max} = 0.5 \). In both cases the unstable mode is the first longitudinal mode of the combustion chamber. This result is in agreement with the experiments in terms of frequency, wave shape and nature of the resonant modes. For a fixed value of the length of the combustion chamber, a reduction of the length of the plenum produces a reduction of the growth rate of the unstable mode whereas the frequency remains constant. Qualitatively this is the same result found during experiments in which a reduction of the amplitude of pressure oscillations is registered once the length of the plenum is reduced. This behavior is proved to be due to an increase of the dissipation of acoustic energy reducing the length of the plenum. It is possible to conclude that, globally, the plenum can be considered to work as a variable geometry Helmholtz resonator. Unlike to a usual Helmholtz resonator [38], the damping effects is active on a wider range of frequencies (∼ 50 Hz). Outside this range, an opposite behavior is registered. So, in order to obtain the wanted dissipation behavior a careful design is needed.

References


“Modelling of thermoacoustic combustion instabilities phenomena: Application to an experimental rig for testing full scale burners”. In ASME Turbo Expo 2014, paper No. GT2014-25273.


